Supplementary Information for McKitrick, McIntyre and Herman, "Panel and Multivariate Methods for Tests of Trend Equivalence in Climate Data Series", Atmospheric Science Letters 2010

1. Further notes on the "effective degrees of freedom" adjustment.

Thanks to Tim Vogelsang for assistance on this discussion.

Suppose regression residuals follow an AR1 process with lag coefficient *r*:

$$u_\tau = r u_{\tau-1} + \varepsilon_\tau$$

where ε_τ is a zero-mean white noise term. Denote the variance of ε_τ as ω^2. Since u_τ is autocorrelated there are non-zero correlations between an observation at time τ and at time $\tau - j$, which we can denote as γ_j The long-run variance of u_τ, denoted σ^2, is defined in the time-series literature as $\gamma_0 + 2\Sigma_j \gamma_j$ and can be shown to equal

$$\sigma^2 = \frac{\omega^2}{(1-r)^2} \qquad (1)$$

The corresponding equation relating γ_j to ω^2 is

$$\gamma_j = r^j \frac{\omega^2}{1-r^2} \qquad (2)$$

Consider *j*=0 and suppose we scale the degrees of freedom term (*T*-2) by $(1-r)/(1+r)$. Via the difference of squares in (2) this yields the following sequence of equalities:

$$\frac{(1+r)}{(1-r)}\frac{1}{T-2}\sum u_\tau^2 = \frac{(1+r)}{(1-r)}\gamma_0 = \frac{(1+r)}{(1-r)}\frac{\omega^2}{(1+r)(1-r)} = \frac{\omega^2}{(1-r)^2} = \sigma^2 .$$

Thus, scaling the degrees of freedom is equivalent, in the AR1 case, to the conventional approach of using (1) to estimate the long-run residual variance. However, this DOF adjustment does not yield the correct residual covariances:

$$\frac{(1+r)}{(1-r)}\frac{1}{T-2}\sum u_\tau u_{\tau-j} = \frac{(1+r)}{(1-r)}\frac{r^j\omega^2}{(1+r)(1-r)} = \frac{r^j\omega^2}{(1-r)^2} \neq \gamma_j .$$

Also, the DOF scaling will not yield the correct residual variance in the presence of higher-order autocorrelation.

2. Interpreting Panel slope estimators

Suppose the trend in y_1 is b_1 and the trend in y_2 is b_2. If the null hypothesis is true, then the trend in y_1 equals the trend in y_2, i.e. $b_1 = b_2$. In the equation

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + d_1 \begin{pmatrix} \tau \\ \tau \end{pmatrix} + d_2 \begin{pmatrix} 0 \\ \tau \end{pmatrix} + \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

the trend in y_1 (i.e. $dy_1 / d\tau$) is identified as d_1 and the trend in y_2 is identified as $d_1 + d_2$. Thus $b_1 = d_1$ and $b_2 = d_1 + d_2$, which implies $d_2 = b_2 - d_1 = b_2 - b_1$. Under the null, $b_1 = b_2$ and the vector $(0\ \ \tau)'$ should have no explanatory power, thus we expect d_2=0. Hence a test of d_2=0 corresponds to a test of $b_2 - b_1$=0.

Note that in the panel regression approach, the variance estimator for d_1 has, in this case, 2*T*-*k* degrees of freedom, and in general *NT*-*k* when there are *N* panels with *T* observations in each. By contrast, the pairwise slope comparisons like Equation (3) in the paper only have than *T*-2 degrees of freedom for estimating the variance. The panel approach is, thus, more efficient. This isn't a trick: under the null hypothesis, observations on y_2 provide as much information about the slope as do observations on y_1, so they should all be counted towards the degrees of freedom.

Text references on panel regressions: D. Gujarati, *Basic Econometrics* 2nd ed., Chapter 14 provides a thorough introduction. See also Kmenta (1986) Sct 11-1; Davidson and MacKinnon (2004) 298—305, both referenced in the paper, for some of the derivations.

3. Derivation of RICH MSU-equivalent data

We were supplied RICH LT and MT MSU-equivalent data generated by L. Haimberger and sent to us via J. Christy. Those data series end in 2006. If we used them as an unbalanced panel in the panel regression model we would obtain the same conclusions as reported in the text, with only minor changes to test scores. Also if we use those data and extrapolate the ending segment using a regression on the other three observational series the VF05 test scores are only trivially changed and our conclusions remain the same.

We deemed it preferable, however, to use the most updated RICH series, which required using the 20N to 20S gridded series and producing our own synthetic MSU series, applying weights supplied by J. Christy.

The scripts are as follows:

http://www.climateaudit.info/scripts/radiosonde/rich.trp.txt

```
#reviewed March 11, 2010
        #new data set ftp://srvx6.img.univie.ac.at/pub/rich_gridded_2009.nc

get.rich.trp=function(method="trp") {
        source("http://www.climateaudit.info/scripts/utilities/getx_var_ncdf.t
xt")
        loc="ftp://srvx6.img.univie.ac.at/pub/rich_gridded_2009.nc"  #formerly
2008
                ###
loc="ftp://srvx6.img.univie.ac.at/pub/rich_gridded_clim_1979-1998.nc" :
climatology
        download.file(loc,"temp.nc",mode="wb")
        v<-open.ncdf("temp.nc")
          length(v$dim)
          names(v$dim) #
                #[1] "lon"      "lat"      "pressure" "time"
          sapply(v$dim,function(A) A$len)
                # lon      lat pressure     time
                # 36       18       12      612
          v$dim$lon$vals # 10 degree Dateline to Dateline
          v$dim$lat$vals # 10 degree N to S
          v$dim$pressure$vals # 850 700 500 400 300 250 200 150 100  70  50
30
          K=length(v$dim$pressure$vals) #12
          v$dim$time$vals # "days since 1958-01-01 00:00:00"         #monthly
          tsp(ts( v$dim$time$vals,start=1958,freq=12) )
            # 1958.000 2008.917   12.000
          names(v$var)
          #anomalies
          sapply(v$var$anomalies,length)
          sapply(v$var$anomalies$dim,function(A) A$len)
                #  36  18  12 612
        instr <- getx.var.ncdf( v, v$var[[1]]) # 1850 2006
           dim(instr) #36  18  12 588
        temp= abs(v$dim$lat$vals)<20;sum(temp) #4
        instr=instr[,temp,,];dim(instr) # 36   4  12 612
        instr=aperm(instr,c(4,1,2,3)); dim(instr) # 588  36   4  12
        dim0=dim(instr)
        instr=array(instr,dim=c(dim0[1],144,12) );dim(instr)
        rich= ts( array (NA,dim=c(dim0[1],12) ),start= 1958,freq=12);tsp(rich)
# 1958.000 2006.917
        for(i in 1:12)  rich[,i]= apply(instr[,,i],1,mean,na.rm=T)
        dimnames(rich)[[2]]= v$dim$pressure$vals # 850 700 500 400 300 250 200
150 100  70  50  30
           dim(rich)
        if(method=="T2LT") {rich_t2lt.trp= apply(rich[,c("850","700")],1,
function(x) weighted.mean(x,c(.33,.67))
 )
          rich_t2lt.trp=ts (rich_t2lt.trp,start=1958,freq=12)

          rich=rich_t2lt.trp}
        return(rich)
}
```

================================ To generate MSU-equivalents using weights supplied by
================================ John Christy:

```
christy.weights=read.table("d:/climate/data/satellite/christy.weights.dat",hea
der=TRUE)
# [1] HadCRU 850     700     500     400     300     250     200     150     100     70
50

load("d:/climate/data/models/santer/Surf_201002.tab")
hadcru= window(Surf$All[,"had"],start=1958,end=2009.99)

library(ncdf)
source("http://www.climateaudit.info/scripts/utilities/getx_var_ncdf.txt")
source("http://www.climateaudit.info/scripts/radiosonde/rich.trp.txt")
rich=get.rich.trp()
dim(rich)
  # 624  12
 tsp(rich)
 # [1] 1958.000 2009.917   12.000
dimnames(rich)[[2]]
         #[1] "850" "700" "500" "400" "300" "250" "200" "150" "100" "70"  "50"
"30"

tsp(rich)
rich=rich[,c(12,1:11)]
rich[,1]=hadcru; dimnames(rich)[[2]][1]="hadcru"

rich.trp = apply( rich,1, function(x)weighted.mean
        (x,christy.weights[,"LT_Weight"]) )
rich.LT=ts(rich.trp,start=1958,freq=12)
        rich.MT=  ts( apply( rich,1, function(x)
weighted.mean(x,christy.weights[,"MT_Weight"]) ),
        start=1958,freq=12)
```

4. Rb=0 as a general form for linear restrictions

Suppose we estimate a regression model

$$y = b_0 + b_1 X_1 + b_2 X_2 + e \,.$$

We want to test the following linear restrictions: (i) $b_1 = b_2$; (ii) $b_1 = b_2 = 0$; (iii) $(b_1 + b_2)/2 = b_0$. They can all be written in the matrix form **Rb=0** as follows. Note $\mathbf{b} = (b_0, b_1, b_2)'$ and $\mathbf{0} = (0,0,0)'$.

$$\text{(i) } \mathbf{R} = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{(ii) } \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{(iii) } \mathbf{R} = \begin{bmatrix} -1 & 0.5 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The rank of **R**, q, is the number of restrictions. Note that q = 2 for (ii) but q = 1 for (iii). In the ‘Obs’ column in Tables 2 and 3, the VF05 tests use q=1, i.e. the test is whether the trend in the mean of the observational series equals the trend in the mean of the modeled series. For the panel regressions, the test is of the same form.

5. Interpretive note on Douglass et al. methodology

Douglass et al. (2007) were not merely testing for model/data agreement in the LT and MT. They added in the constraint that the models have to agree at the surface. That is, they shift the vertical trend profile as needed so that the surface trend agrees with observations. The reasoning is that it would overstate the fidelity of models to data in the troposphere if agreement aloft only ever accompanied a prediction of no warming or cooling at the surface while the observations show warming at the surface.