Carbon Taxes and Damage Thresholds in the Presence of Pre-Existing Regulations

Ross McKitrick
Department of Economics and Finance
University of Guelph
ross.mckitrick@uoguelph.ca

March 13, 2018

Abstract: Many studies have looked at the welfare effects of a carbon tax in the absence of regulation and vice versa. The welfare implications when one is layered on top of the other have been implicit in models of tax interactions but not widely discussed, even though the policy practice is commonly observed. I examine this issue in a model of multiple polluting sectors under a second-best tax system. As has been noted previously, policy interactions can give rise to a value threshold whereby, if marginal damages are positive but below it, any emission reduction policy is welfare-reducing. This is a striking challenge to conventional intuition about pollution policy but it has received relatively little discussion. I characterize the damage threshold and show that it can occur under more general conditions than previously noted. Partial regulation of emitters prior to introduction of an carbon tax raises the threshold.

Key words: emissions taxes, tax interactions, damage thresholds, climate policy

JEL Codes: H21, H23, Q54, Q58

Acknowledgments: I am grateful to Don Fullerton and Les Shiell for helpful comments.

1 INTRODUCTION

Emission taxes interact with the rest of the tax system in two key ways. The revenues can be "recycled" to reduce other taxes, thereby yielding welfare gains over and above the pollution reductions. But the new charges on pollution-intensive goods also increase the marginal distortions in the pre-existing tax system, especially by reducing real wages and the labour supply. A large literature has shown that the latter "tax interaction effect" typically dominates the revenue recycling effect, with the implication that the optimal pollution tax should be lower than marginal damages. Sandmo (1975), Bovenberg and Goulder (1996), Parry et al. (1999), Schöb (2003) and others solved models in which the ratio of the optimal emissions tax to marginal damages equals the inverse of the marginal cost of public funds,[1] a result I refer to herein as the Sandmo rule. It implies that the optimal pollution tax is a (below-unity) scalar multiple of marginal damages, so as long as marginal damages are positive, a sufficiently-small pollution tax must be welfare-improving.

Numerical simulations in Bovenberg and Goulder (1996), Goulder et al. (1997) and Bento and Jacobsen (2007) showed, however, that the optimal tax is not always merely a multiple of marginal damages but can include an additive term as well. Or, re-expressing the same thing, the marginal social cost of abatement is a transformation of the marginal private cost, involving both a rotation upwards (by a factor corresponding to the marginal cost of public funds) and a shift, so that the first unit of emissions reductions has a discrete positive cost. If marginal damages are positive but below this threshold, any emission reduction, whether by price or quantity instrument, is welfare-reducing.

---

[1] The marginal cost of public funds is the dollar-valued welfare cost of raising an additional dollar for government spending. Unless the tax system is first-best (non-distorting) it exceeds unity.

This striking result arose in numerical simulations of non-revenue raising instruments, or revenue-raising instruments with the revenues returned in a lump-sum fashion, and also when the emissions tax was imposed on an economy in the presence of an above-optimal income tax rate. Bovenberg and Goulder (1996) and Goulder et al. (1997) provided numerical estimates that emphasized its policy significance. Under reasonable assumptions about parameter values, marginal damages would have to be relatively high in the case of carbon dioxide (over $50 US per ton in 1996 dollars) and sulfur dioxide (over $100 per ton in 1997 dollars) for any non-revenue raising policy to be welfare-improving. This point has received only limited attention in the academic literature on environmental policy and almost none in the popular and applied discussions of carbon pricing, despite its obvious salience for climate policy.[2] Subsequent theoretical work yielded additional insights into the possibility of a positive damage threshold, confirming that its existence and magnitude is sensitive to policy design. Parry et al. (1999), Goulder (1998), and Goulder (2013) all showed that it chiefly arises under non-revenue raising policies, although Fullerton and Metcalf (2001) argued that the key mechanism is not revenue-raising *per se* but whether the policy creates scarcity rents that are left in private hands. In their model, if another fiscal instrument captures the rents and uses them to offset tax interactions then the threshold disappears.

---

[2] For instance, Gregory Mankiw's famous 2006 blog post calling for higher gasoline taxes http://gregmankiw.blogspot.ca/2006/10/alternatives-to-pigou-club.html lists (and rejects) what he sees as the only four reasons why an economist might oppose raising externality taxes, ignoring the possibility of a positive damage threshold.

The first goal of this paper is to set up and solve a model of second-best emission taxes that ties together the previous work, showing in a tractable way the circumstances in which a positive damage threshold arises, and providing a theoretical explanation of results that have emerged from numerical simulations. The second purpose is to extend the literature on damage thresholds to look at a policy configuration common in practice but unexamined in theory: the imposition of carbon taxes in economies where some sectors are already regulated. Many studies have compared policies on an either/or basis: either emission taxes or quota-type regulations, but not both simultaneously. In practice, however, especially regarding carbon taxes, layering emission charges on top of pre-existing regulations has been the rule rather than the exception. While a few commentators[3] have pointed out that economic theory calls for a policy swap rather than a combination, many others explicitly call for carbon taxes to be used in combination with other regulatory measures (for example, Stavins 2015, Fankhauser 2012).[4] And in practice such an approach has been ubiquitous when emission pricing is introduced. For example, in 2017 the Government of Canada announced a minimum national carbon price of ten dollars per tonne to begin in 2018, rising to $50 per tonne by 2022, which provincial governments must implement either as a tax or a tradable permit system.[5] But Canadian federal and provincial governments have already implemented numerous sectoral

[3] For example, Clemens and Green (2017), Taylor (2015).

[4] Fischer et al. (2017) look at a sectoral model of electricity where there are multiple distinct market failures, including knowledge spillovers and CO_2 emissions, hence multiple policy instruments are required. This differs from the present case in which multiple instruments are applied on a single market failure.

[5] See Government of Canada website http://news.gc+.ca/web/article-en.do?nid=1132149 accessed May 4, 2017.

carbon regulations, such as coal phase-outs in Ontario and Alberta, a hard cap on carbon dioxide emissions from the Alberta oil sands, national ethanol blending requirements in gasoline, etc., none of which are to be repealed as a result of the introduction of the pricing requirement. Similarly, carbon permit trading systems in California, the European Union and elsewhere operate in addition to, not instead of, regulations that aim to limit carbon dioxide emissions.

Though the case has not been studied directly, the literature discussed above provides strong intuition that pre-existing regulations will inflate the marginal welfare cost of the new emission taxes, in much the same way that pre-existing taxes would. I confirm this intuition in the model developed below. The literature also hints at the possibility that, by creating (or increasing) a damage threshold, pre-existing regulations can destroy the conditions in which a carbon tax can be welfare-improving, but the question has never formally been asked. I model this situation herein and confirm the conjecture.

I study an economy with two polluting firms and a government sector funded by a labour tax and derive the optimal second-best emissions tax. I then look at a case in which one firm is subjected to a binding emission constraint prior to an economy-wide emission tax being introduced. Welfare is unambiguously lower compared to the optimal tax case without partial regulations. The regulations increase the damage threshold, so that even if it was zero previously it will be positive in the presence of the regulations.

This result, and the extensive literature preceding it, has important implications for policy discussions around emission pricing and carbon taxes, especially in the presence of pre-existing regulations that are not simultaneously being repealed. An emissions tax equal to marginal damages is only guaranteed to improve welfare under restrictive conditions that are easily violated in real-world economies. Under more general conditions, the pricing rule for marginal damages (or the Social Cost of Carbon (SCC) as it is called in climate policy) is more complex and the potential welfare effects need to be carefully qualified.

The next section provides a brief graphical explanation of the main theoretical results. Section 3 sets up the model and explains the results previously shown in numerical simulations. Section 4 looks at the partial regulation case and Section 5 offers discussion and conclusions.

2 GRAPHICAL SUMMARY

In Figure 1 the line labeled MD represents marginal damages of emissions, which are assumed to be constant. The line labeled MAC_p denotes *private* marginal abatement costs, or marginal profits of emissions, and the horizontal intercept $\bar{E}$ is the unregulated (privately optimal) emissions level. The classical Pigovian emissions tax, where $MD = MAC_P$, is labeled τ_P. The associated optimal emissions level is denoted by E_P. The line MAC_s denotes social marginal abatement costs, namely MAC_p plus the welfare costs of tax interaction effects net of revenue-recycling benefits. The optimal emissions level is where $MD = MAC_s$ which is at E_1, and since firms respond to a tax according to MAC_p, the corresponding optimal price is τ_1, which according to the Sandmo rule is aMD where $a = MCPF^{-1}$. The rotation of the MAC_P curve still implies that the social costs of the policy go to zero as the emission fee goes to zero.

Figure 2 illustrates the threshold case. MAC_p undergoes both a rotation and a translation out to the new MAC_s. The shift to the right of the horizontal intercept arises when it becomes possible to improve welfare via a directional tax reform that includes a subsidy to emitting activity. If negative pollution taxes are ruled out then the maximum emissions level is $\bar{E}$ and the first unit of emission reductions has a discrete social cost of $Z > 0$. If the relationship between MD and the optimal emissions tax is given by $\tau = aMD - b$ where a and b are both positive constants, solving for the value of MD consistent with an optimal emissions tax of zero yields $Z = b/a$. As drawn, $Z > MD$ so the emissions policy is unambiguously welfare-reducing even though MD is positive. If $MD > Z$ an emissions policy is still warranted but the effect of the threshold on the gap between MAC_p and MAC_s needs to be taken into account when setting the optimal tax rate. It will be shown below that if regulations are introduced that restrict emissions from some sectors, Z increases. The same occurs if the tax on labour is raised above its optimal level.

3 DAMAGE THRESHOLD MODEL

3.1 MODEL SET-UP

There are N identical households, two goods each produced by a separate firm, an energy sector, a labour market and a government. Household-specific goods consumption is denoted x_i, $i = (1,2)$, the corresponding prices are p_i, and aggregate demand is denoted $X_i = Nx_i$. Households also consume e_{hh} units of energy at price p_E and aggregate household energy demand is $E_{hh} = Ne_{hh}$. Households each have a time endowment t which can be allocated to labour l or leisure h, so the aggregate labour supply is $L = Nl$, aggregate leisure is $H = Nh$ and the aggregate time endowment is $T = Nt$. The before-tax nominal wage rate is w. We will set w as the numeraire so it is constant and equal to unity but for notational clarity I retain it in the derivations.

Energy is produced by a sector that uses only labour L_E (or equivalently that does not need to purchase energy outside itself)

$$E = F^E(L_E) \tag{1}$$

where E is the total energy supply. Energy sector profits are

$$\pi^E = p_E F^E(L_E) - wL_E \tag{2}$$

The first order condition implies $F_L^E = w/p_E$. Use of energy causes emissions of carbon dioxide according to the function $C = cE$ where c denotes the emissions intensity of energy. Emissions are taxed at τ_C per unit so the tax-inclusive price of energy is

$$p_E' = p_E + c\tau_C. \tag{3}$$

Note that $'$ throughout denotes a tax-inclusive term.

Each goods-producing firm has a single unit of fixed capital K_i equal to unity and a production function $F^i(L_i, E_i)K_i$ where the first argument denotes labour usage and the second denotes energy. As in Bento and Jacobsen (2007) assume that F^i is strictly concave and has decreasing returns to scale in L_i and E_i. The profit function for firm *i* is

$$\pi_i = p_i F^i(L_i, E_i) - wL_i - p_E' E_i \tag{4}$$

where $F_L^i > 0$ and $F_E^i > 0$. The first-order conditions imply $F_L^i = w/p_i$ and $F_E^i = p_E'/p_i$. Decreasing returns to scale imply that profits are positive and represent the return to capital for each firm. We assume shares in firms are distributed equally among all households.

We will assume that prices are initially normalized so that $p_i = p_E = w = 1$. The tax rate on household income is τ_Y and net income is

$$y' = \left(\frac{\pi_1+\pi_2+\pi_E}{N} + wt\right)(1-\tau_Y). \qquad (5)$$

The household budget constraint is

$$p_1 x_1 + p_2 x_2 + p_E' e_{hh} + w'h = y' \qquad (6)$$

where $w' = w(1-\tau_Y)$. The corresponding national budget constraint (NBC) is

$$p_1 X_1 + p_2 X_2 + p_E' E_{hh} + w'H = Y' \qquad (7)$$

where $Y' = (\pi_1 + \pi_2 + \pi_E + wT)(1-\tau_Y)$.

The government does not use energy but purchases some of the available production of good 2 and gives it to households in equal shares. It finances this through the tax τ_C on emissions C and the income tax τ_Y. Hence the Government Budget Constraint (GBC) is

$$p_2 G = \tau_Y B + \tau_C C \qquad (8)$$

where the income tax base B equals $\pi + wL$ and $\pi = \pi_1 + \pi_2 + \pi_E$.

Goods Market Equilibrium (GME) occurs where $X_1 = F_1$ and $X_2 + G = F_2$. Energy Market Equilibrium (EME) occurs where $E_{hh} + E_1 + E_2 = E$. Labour Market Equilibrium (LME) occurs where $L_1 + L_2 + L_E = T - H$. I confirm in the Appendix that imposing GME, LME and EME on the NBC implies the GBC holds; likewise any four implies the fifth.

We assume that tax rates are adjusted to hold G constant. Differentiating Equation (8) and rearranging yields the G-neutrality tax condition

$$\frac{d\tau_Y}{d\tau_C} = \frac{1}{B}\left(G\frac{dp_2}{d\tau_C} - \tau_Y\frac{dB}{d\tau_C} - \tau_C\frac{dC}{d\tau_C} - C\right). \qquad (9)$$

Since the emissions tax raises the cost of providing the public good and causes the income tax base and emissions to decline, the first three terms in the brackets must sum to a positive number. We will assume that we are operating in a region of the economy for which the new tax revenue (represented by the fourth term) is sufficiently large as to make whole derivative negative, meaning that an increase in emission taxes permits a reduction in the income tax.

Household utility is $u(x_1, x_2, e_h, h) + \frac{\alpha G}{N} - \delta C$ where α is the positive welfare weight on the public good G and δ is the marginal welfare cost of each unit of emissions. We will use the indirect utility function $v(p_1, p_2, p_E, w', y')$ to define the national social welfare function

$$W = Nv(p_1, p_2, p_E', w', y') + \alpha G - \delta NC. \qquad (10)$$

The planner's problem is to choose τ_C to maximize (10). Since G is assumed fixed and given, τ_Y is then determined by equation (8).

3.2 Derivation of Optimal Taxes

The first derivative of equation (10) with respect to τ_C is

$$\frac{dW}{d\tau_C} = N\left(v_1\frac{dp_1}{d\tau_C} + v_2\frac{dp_2}{d\tau_C} + v_E\frac{dp_E'}{d\tau_C} + \; v_w\frac{dw'}{d\tau_C} + v_y\frac{dy'}{d\tau_C}\right) - \delta N\frac{dC}{d\tau_C} \qquad (11)$$

where first derivatives of v are subscripted in order of the arguments. Divide equation (11) by v_y and apply Roy's theorem to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - E_{hh}\frac{dp_E'}{d\tau_C} - H\frac{dw'}{d\tau_C} + \frac{dY'}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}. \qquad (12)$$

Note that $\frac{dY'}{d\tau_C} = (1-\tau_Y)\frac{d\pi}{d\tau_C} - \pi\frac{d\tau_Y}{d\tau_C} + H\frac{dw'}{d\tau_C} + L\frac{dw'}{d\tau_C}$. In the Appendix I show that this expression combined with equations (9) and (12) yield

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = \frac{dC}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right) - Q + \tau_Y R \qquad (13)$$

where

$$Q = \left(F^1\frac{dp_1}{d\tau_C} + F^2\frac{dp_2}{d\tau_C}\right) - \tau_Y w\left(\frac{\partial L}{\partial p_E'}c + \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_C} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_C}\right) \qquad (14)$$

and

$$R = -w^2\frac{\partial L}{\partial w'}\frac{\partial \tau_Y}{\partial \tau_C}. \qquad (15)$$

We can decompose equation (13) into some standard components (compare to Bento and Jacobsen 2007, Parry et al. 1999 and others). The first term on the right side of equation (13) is the primary welfare effect of the emissions tax, represented by the change in emissions times the difference between the tax τ_C and $\frac{\delta N}{v_y}$, which is the marginal external cost of emissions, or MD. The equation for Q contains two parts that together represent the tax interaction effect, namely the welfare costs resulting from price changes induced by the emissions tax. The first term measures costs to households and the government of the increased output prices resulting from the emissions tax. The second term represents the lost tax revenue due to the labour market effects of the emissions tax. On the supply side, an increase in prices reduces real wages and thus reduces labour supply. On the demand side, under strict concavity each firm's demand for labour must decline when the price of energy rises (Henderson and Quandt 1980 p. 81). Consequently, the change in L must be negative. Combining these yields $Q > 0$. $\tau_Y R$ represents the offsetting increase in labour tax revenue when the emission tax finances a reduction in the income tax rate, which induces an increase in the labour supply. Examination of the derivatives shows $R > 0$.

In the Sandmo (1975) framework, if the government revenue requirement is low enough to be fully satisfied by the externality tax, the optimal policy would entail a tax on the dirty good equal to marginal social damages and no other tax. That outcome does not emerge here, however. If we set $\tau_Y = 0$ then setting equation (13) equal to zero yields

$$\tau_C = \frac{\delta N}{v_y} + \frac{F^1 \frac{dp_1}{d\tau_C} + F^2 \frac{dp_2}{d\tau_C}}{dC/d\tau_C}$$

which is strictly less than *MD*. The reason for the difference is that the model herein allows producer prices to change whereas in the Sandmo framework they are fully determined by fixed input-output coefficients. If prices were similarly fixed herein the second term would disappear and the classical solution would emerge.

If we imposed constant returns to scale, as in Fullerton and Metcalf (1997), Parry et al. (1999), and most others, some important differences would arise. The price derivatives would take the form $\frac{dp_i}{d\tau_C} = \frac{E_i}{F^i}$ (see derivation in the Appendix A, Parry et al. 1999) and profits would be identically zero. Hence the income tax base B would consist only of labour income, so the tax optimization problem would reduce to a choice of relative tax burdens on each of the two inputs to production. Since neither tax base would encompass the other there would be no unambiguous efficiency differences and the tax interaction and revenue recycling effects would follow from arbitrary assumptions about market demand elasticities. Parry et al. (1999) include three production inputs (labour, a clean good and a dirty good) so the labour tax has a broader base than that on the dirty good, which creates a difference in relative distortions independent of the market elasticities.

It is common to derive the optimal emissions tax from equation (13) by setting it equal to zero and rearranging to $\tau_C = MD + Q/\frac{dC}{d\tau_C} - \tau_Y R/\frac{dC}{d\tau_C}$ (see, e.g. Bento and Jacobsen 2007 equation 2.12). This is an incomplete solution since the income tax rate is determined by τ_C. In the Appendix I show that using the GBC to substitute out τ_Y then solving for the optimum yields

$$\tau_C^* = aMD - b \qquad (16)$$

where

$$a = \frac{\frac{dC}{d\tau_C}}{\frac{dC}{d\tau_C} - \frac{RC}{B}} \tag{17}$$

and

$$b = a\left(\frac{p_2 G}{B} R - Q\right)\left(\frac{dC}{d\tau_C}\right)^{-1} \tag{18}$$

Since $R > 0$ and $dC/d\tau_C < 0$, the weight a on MD must be positive and less than unity. In the Appendix I present a discussion of its relation to the marginal cost of public funds. To sign b we need to sign the term in the brackets. Note that R/Q is the ratio of the revenue recycling benefits to the tax interaction losses. If this equals B/P_2G, which measures the ratio of income to the government budget, then $b = 0$. Previous empirical work has found that R/Q is typically less than unity: the tax interaction effect outweighs the revenue recycling benefit yielding a ratio of about 0.7 (Parry 1995). Also the ratio of the tax base to government spending must be greater than unity, and in this case is approximately the inverse of the income tax rate. Thus $B/P_2G > R/Q$ which implies $Q > RP_2G/B$ and therefore $b > 0$. By completing the derivation it becomes clear that $\tau_c^* < MD$.

Equation (16) can be rendered into the same terms as Figures 1 and 2 by noting that MAC_p corresponds to τ_C^* and the social optimum occurs where $MAC_s = MD$, so rearranging yields

$$MAC_s = \frac{1}{a} MAC_p + \frac{b}{a}$$

making clear that the difference between the two can involve both a rotation and a shift upwards.

3.3 BENEFITS THRESHOLD

As noted previously, we can derive an expression for the threshold value by solving equation (16) for MD assuming $\tau_C = 0$. We obtain $Z = \frac{b}{a}$ which is positive. This result can occur even in a second-best optimal setting with a revenue-raising instrument, a result not previously shown in models with fixed prices and/or constant returns to scale. Numerical simulations in Bovenberg and Goulder (1996) and others showed that Z is non-negative, but the magnitude strongly depends on the form of the policy.

Non-revenue raising policy

Suppose that the policy is non-revenue raising, such as in the case of CAC or tradable quotas. Assume emissions are restricted to $\hat{C}$ which has an associated shadow price which we will call $\hat{\tau}_C$, which is the marginal value to the firm of being allowed to increase emissions by one unit, which corresponds to the firms' private marginal abatement costs MAC_p. We will assume this price is the same for each firm, as would happen under tradable quotas. The planner's problem can be re-stated as an optimization of W by choice of $\hat{C}$. In the Appendix I show that the solution occurs at

$$MAC_p = MD + \hat{Q} - \tau_Y \hat{R} \tag{19}$$

where $\hat{Q} = F^1 \frac{dp_1}{d\hat{C}} + F^2 \frac{dp_2}{d\hat{C}} - w\tau_Y \left(\frac{\partial L}{\partial p_E'} \frac{\partial p_E'}{\partial \hat{C}} + \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial \hat{C}} + \frac{\partial L}{\partial p_2} \frac{\partial p_2}{\partial \hat{C}} \right)$ and $\hat{R} = w \frac{\partial L}{\partial w'} \frac{\partial w'}{\partial \hat{C}}$. Note the signs of derivatives here are different. There is now only one tax in the economy (τ_Y), the derivative of $\hat{C}$ with respect to τ_C does not appear and there is no weighting coefficient on MD. An increase in $\hat{C}$

reduces costs for firms, so $\hat{Q} < 0$. Since an increase in allowed emissions (as opposed to a change in the relative cost of labour and emissions) raises employment we have $\hat{R} > 0$. Hence the solution occurs where $MAC_p < MD$.

The benefits threshold is found by setting Equation (19) equal to zero and solving for MD, yielding $\hat{Z} = -\hat{Q} + \tau_Y \hat{R}$. Even if $\hat{Q} = 0$ at the unregulated emissions level (the first unit of abatement is costless to the firm) $\hat{R}$ will be positive because the labour supply elasticity is positive and emission constraints shrink the tax base, forcing up τ_Y to maintain $dG = 0$. Hence $\hat{Z}$ is unambiguously positive for a non-revenue raising policy, as noted in Goulder et al. 1997, Parry et al. 1999 and Fullerton and Metcalf 2001. We note additionally from equation (19) that the threshold will be larger, the higher is the slope of the labour supply function and the price responses to changes in $\hat{C}$. The overall implication is that, even if MD is positive, a reduction in emissions below the unregulated amount through a non-revenue raising policy will reduce welfare unless MD exceeds a positive threshold. Even when it does exceed the threshold the optimal emissions level occurs at $MAC_s = MD$, which exceeds the conventional textbook level where $MAC_p = MD$. This is a pretty fundamental revision of the conventional textbook model, yet despite the widespread use of quantity regulations it remains unnoticed outside the specialist literature.

Non-optimal tax system

Suppose we use a pricing instrument but the rest of the tax system is sub-optimal. This case was noted in Bovenberg and Goulder (1996), in which the optimal pollution tax only corresponded to the Sandmo result ($MD/MCPF$) when the tax system was (second-best) optimized. It can be demonstrated in the current context by returning to equation (13) but instead of setting it equal to zero, set it equal to some other value ρ:

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = \frac{dC}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right) - Q + \tau_Y R = \rho \tag{20}$$

The non-optimality we are interested in involves an overly-high income tax τ_Y, which under the GBC implies a lower emissions tax. The solution to equation (20) is now

$$\tau_C^{\rho} = aMD - b + a\rho\left(\frac{\partial C}{\partial \tau_C}\right)^{-1} \tag{21}$$

where a and b are defined as before. Comparing this to equation (16) we see that the difference between τ_C^{ρ} and τ_C^{*} is the third term on the right. Since the welfare function is convex, $\tau_C^{\rho} < \tau_C^{*}$ implies a positive value of ρ. We also know $a > 0$ from before and the derivative is negative, so the third term could yield a negative optimal emission tax depending on the magnitude of ρ.

The benefits threshold is now

$$Z^{\rho} = \frac{b}{a} - \frac{\rho}{dC/d\tau_C} \tag{22}$$

Since the second term is positive we have $Z^{\rho} > Z$, or in other words when the tax system is not optimized, such that labour is over-taxed, the benefits threshold is unambiguously raised. Even if the benefits threshold in the optimized case is zero, so that the first unit of abatement under an emission pricing policy is welfare-enhancing, the existence of a non-optimal labour tax is sufficient to create a positive threshold for MD.

4 Emission Taxes Under Pre-Existing Regulations

We now turn to the case in which pre-existing command-and-control regulations cover some, but not all, emissions. The outcome under this case is denoted with ~. Suppose that firm 2 is required to reduce emissions to a fixed target level $c\tilde{E}_2$ which is below the level associated with any proposed emissions tax rate.[6] The shadow price associated with the emissions constraint is denoted $\tilde{\tau}_C$, which must lie above the emissions tax rate τ_C by assumption. Both firms pay the tax but only firm 1 freely chooses its emissions level. Consequently, the first order conditions remain the same for firm 1, but for firm 2 only that related to labour remains the same. The profit function for firm 2 is now

$$\pi_2 = p_2 F^2\left(L_2, \tilde{E}_2\right) - wL_2 - (p_E + \tau_C c)\tilde{E}_2.$$

The emissions charge is a lump-sum rent capture for firm 2, which will help finance beneficial tax reductions, but the regulation itself restricts the production function and results in an upward shift of its supply curve, by an amount exceeding what would have been experienced under the emission tax. The price charged under the regulation is denoted $\tilde{p}_2$ and is strictly greater than p_2. Firm 2 profits $\tilde{\pi}_2$ are also lower than in the previous case. The regulation does not change the relative price

[6] If $\tilde{E}_2$ exceeded this level then the constraint would not bind and we would simply be back to the case of a uniform emissions tax across both sectors.

of the inputs, instead it forces down output and profitability so it has a negative effect on labour demand, leading to $\tilde{L}_2 < L_2$. Since profits and labour earnings are reduced we expect $\tilde{y}' < y'$. These changes make it unambiguous that $v(p_1, \tilde{p}_2, p_E, \widetilde{w}', \tilde{y}') < v(p_1, p_2, p_E, w', y')$. Also the tax base shrinks, i.e. $\tilde{B} < B$.

Denote the total level of emissions in this case as $\tilde{C} = C_1 + \tilde{C}_2$ which is strictly less than C. Because both firms pay the emissions tax we have $\frac{d\tilde{\pi}}{d\tau_C} = -cE_1 - c\tilde{E}_2$. The NBC is $p_1X_1 + \tilde{p}_2X_2 + (p_E + \tau_C c)E_{hh} + \widetilde{w}'H = (\pi_1 + \tilde{\pi}_2 + wT)(1 - \tau_Y)$. The GBC is $\tau_C\tilde{C} + \tau_Y\tilde{B} = \tilde{p}_2G$. Since the emissions level is fixed for firm 2, $\frac{d\tilde{p}_2}{d\tau_C} = 0$ and the G-neutrality condition is written

$$-\tilde{B}\frac{d\tau_Y}{d\tau_C} = \tau_C\frac{dC_1}{d\tau_C} + \tilde{C} + \tau_Y\frac{d\tilde{B}}{d\tau_C}.$$

The derivative of W with respect to τ_C looks like Equation (12) with C_1 replacing C:

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - E_{hh} - H\frac{dw'}{d\tau_C} + \frac{d\tilde{Y}'}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}.$$

The form of $\frac{d\tilde{Y}'}{d\tau_E}$ is unchanged from before and the derivation proceeds in the same way. In the Appendix I show that the optimal tax rate is now

$$\tilde{\tau}_C = \tilde{a}\frac{\delta N}{v_Y} - \tilde{b} \tag{23}$$

where

$$\tilde{a} = \frac{\frac{dC_1}{d\tau_C}}{\frac{dC_1}{d\tau_C} - \frac{\tilde{R}\tilde{C}}{\tilde{B}}}$$

$$\tilde{b} = \tilde{a}\left(\frac{p_2 G}{\tilde{B}}\tilde{R} - \tilde{Q}\right)\left(\frac{dC_1}{d\tau_C}\right)^{-1}$$

$$\tilde{Q} = F_1 \frac{dp_1}{d\tau_C} - \tau_Y w \left(\frac{\partial L_1}{\partial p_E'} c + \frac{\partial L_1}{\partial p_1}\frac{\partial p_1}{\partial \tau_C}\right)$$

and

$$\tilde{R} = \tau_Y w \frac{\partial L_1}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}.$$

Since the tax base has shrunk and the cost of government spending has risen, either or both of the income and emission taxes must be higher compared to the no-regulation case.

Due to the number of simultaneous changes in magnitudes, it is not straightforward to compare the old and new thresholds. In order to do so we will make use of some ratio terms. Denote by r_1 the fraction of the economy not subject to regulation, in other words sector 1 plus the energy sector as a fraction of total output. In very rough terms, we might expect about a third of the economy to be subject to regulations, so $r_1 \cong 0.7$ (though the precise value does not matter). This is the factor that will apply to R and Q. Denote by r_2 the shrinkage of the tax base as a result of the regulations. Even for very costly regulations we would not expect a large recession to be induced, so this parameter would be close to unity (for example, $r_2 = 0.99$). Finally denote the increase in the cost of government spending by r_3, which equals the increase in p_2 due to the regulation. Again, even for stringent regulations this will not typically go above unity by much, so assume $r_3 = 1.1$.

Following the usual derivation, we have

$$\tilde{Z} = \frac{\tilde{b}}{\tilde{a}} = \frac{1}{\frac{dC_1}{d\tau_C}}\left(\frac{\tilde{p}_2 G\tilde{R}}{\tilde{B}} - \tilde{Q}\right)$$

To compare this with Z, apply the ratio terms, obtaining

$$\tilde{Z} \cong \frac{1}{r_1 \frac{dC}{d\tau_C}}\left(\frac{r_3 p_2 G r_1 R}{r_2 B} - r_1 Q\right) = \frac{1}{\frac{dC}{d\tau_C}}\left(\frac{p_2 GR}{B} \times \frac{r_3}{r_2} - Q\right) \tag{24}$$

The difference between $\tilde{Z}$ and Z is the ratio r_3/r_2 in the brackets, which can be thought of as an index of the impact of the regulation. The index measures the increased cost of funding the government and shrinkage of the tax base due to the regulation, and it must be greater than unity, hence $\tilde{Z}$ must be greater than Z. Consequently, the partial regulation raises the damage threshold, and the more stringent the regulation, the larger the effect. Pre-existing regulations thus undermine the potential for emission taxes to be welfare-improving. Even if the damage threshold attributable to tax interactions were zero, once emissions are constrained by partial regulations, a positive threshold will exist and may create the condition that an emission tax of any magnitude is welfare-reducing.

5 DISCUSSION AND CONCLUSIONS

This paper has examined the design of an optimal emission tax in an economy where pollution generates positive marginal damages. Previous literature has shown that the optimal emission fee is of the form $\tau = aMD - b$ where $a < 1$ is a commonly-derived result and $b > 0$ is possible but not guaranteed. The model developed herein is somewhat more general than that of earlier literature, allowing two polluting sectors, decreasing returns to scale and variable prices. I derive a second-best optimum in which $a < 1$ and $b \neq 0$ and discuss the assumptions that drive this result. $b = 0$ would only occur if the tax interaction and revenue-recycling effects balance in a particular way, or strong assumptions are imposed such as fixed prices. Otherwise the stylized empirical facts suggest we would expect to observe $b > 0$, implying a positive damage threshold.

I then consider a case which has been overlooked in previous analyses but which is ubiquitous in practice, in which emissions are partially regulated prior to an emission tax being introduced. The regulation reduces the welfare of the starting equilibrium relative to the pure pricing case. The stringency of the regulation affects prices and the size of the income tax base, and the ratio of these two factors determines the size of the increase in the damage threshold.

The overall implication of this modeling framework is that tax interactions not only imply that the optimal emissions tax should be less than marginal damages by a factor reflecting the marginal cost of public funds, but that there may be a minimal threshold for damages below which any emissions tax is welfare-reducing even when marginal damages are positive. This threshold value is unambiguously positive when an emissions control policy is non-revenue raising, which likewise implies that the first unit of command-and-control regulation may be welfare-reducing if marginal damages are positive but below the threshold. It is increased when the labour tax is above the optimal rate and when emitters are partially regulated prior to the tax being introduced.

These findings represent an important refinement to the standard treatment of optimal emission policy in most environmental economics textbooks, which assume an equivalence between private and social marginal abatement costs.[7] Real-world divergences between these two have direct implications for, among others, ongoing debates about climate policy. It is notable how little work has been done to elucidate the conditions that give rise to a damage threshold and how little empirical information has been generated about its magnitude. Bovenberg and Goulder (1996) estimated a threshold for non-revenue raising policies (like tradable quotas) of about US$55, which would exceed US$75 per tonne in 2017 dollars. Mainstream estimates of the social cost of carbon are well below this latter amount (Interagency Working Group 2013), implying that any emission abatement regulation (including non-auctioned tradable quotas) is welfare-reducing. The size of the threshold effect created by partial sectoral regulations also needs to be examined empirically, a topic which is left for subsequent work.

6 REFERENCES

Bento, Antonio and Mark Jacobsen (2007) "Ricardian Rents, Environmental Policy and the 'Double-Dividend' Hypothesis." *Journal of Environmental Economics and Management* 53 pp. 17—31.

Bovenberg, A. Lans and Ruud A. de Mooij (1994) "Environmental Levies and Distortionary Taxation" *The American Economic Review* Vol. 84, No. 4 (Sep., 1994), pp. 1085-1089

Bovenberg, A. Lans and Lawrence H. Goulder (1996). "Optimal Environmental Taxation in the Presence of Other Taxes: General-Equilibrium Analyses." *American Economic Review* 86(4) 985—1000.

Clemens, Jason and Kenneth Green (2017) "The Myths and Realities of Carbon Pricing in Canada and beyond." *The Globe and Mail* March 6, 2017.

Fankhauser, Samuel (2012) "A Practitioner's Guide to a Low-Carbon Economy: Lessons from the UK." Centre for Climate Change Economics and Policy, Grantham Research Institute on Climate

[7] McKitrick (2010) chapter 8 is an exception.

Change and the Environment Policy Paper January 2012, available online at http://www.lse.ac.uk/GranthamInstitute/wp-content/uploads/2014/03/PP_low-carbon-economy-UK.pdf

Fischer, Carolyn, Louis D. Preonas, and Richard G. Newell (2017) "Environmental and Technology Policy Options in the Electricity Sector: Are We Deploying Too Many?," *Journal of the Association of Environmental and Resource Economists* https://doi.org/10.1086/692507

Fullerton, Don and Gilbert E. Metcalf (2001) "Environmental Controls, Scarcity Rents and Pre-Existing Distortions." *Journal of Public Economics* 80 pp. 249—267.

Goulder, Lawrence H. (1998) "Environmental Policy Making in a Second-Best Setting." *Journal of Applied Economics* I(2) November 1998, 279—328.

Goulder, Lawrence H. (2013) "Climate Change Policy's Interactions with the Tax System." *Energy Economics* http://dx.doi.org/10.1016/j.eneco.2013.09.017

McKitrick, Ross R. (2010) *Economic Analysis of Environmental Policy*. Toronto: University of Toronto Press.

Metcalf, Gilbert (2003) "Environmental Levies and Distortionary Taxation: Pigou, Taxation and Pollution." *Journal of Public Economics* 87 (2003) 313-322.

Parry, Ian W.H. (1995) "Pollution taxes and revenue-recycling" *Journal of Environmental Economics and Management* 29 S64-S77.

Parry, Ian W.H. (1997). "Environmental taxes and quotas in the presence of distorting taxes in factor markets." *Resource and Energy Economics* (19), 203–220.

Parry, Ian, Roberton C. Williams III and Lawrence H. Goulder (1999). "When Can Carbon Abatement Policies Increase Welfare? The Fundamental Role of Distorted Factor Markets." *Journal of Environmental Economics and Management* 37: 52—84.

Sandmo, Agnar (1975) "Optimal taxation in the presence of externalities." *Swedish Journal of Economics* 77 (1), 86–98.

Schöb, Ronnie (2003) "The Double Dividend Hypothesis of Environmental Taxes: A Survey." Fondazione Eni Enrico Mattei Working Paper No. 60.2003, 11 June 2003 http://papers.ssrn.com/abstract=413866

Stavins, Robert (2015) "Both Are Necessary, But Neither is Sufficient: Carbon-Pricing and Technology R&D Initiatives in a Meaningful National Climate Policy." Weblog posting available online at http://www.robertstavinsblog.org/2010/10/21/both-are-necessary-but-neither-is-sufficient-carbon-pricing-and-technology-rd-initiatives-in-a-meaningful-national-climate-policy/

Taylor, Jerry (2015) "The Conservative Case for a Carbon Tax." Niskanen Centre Report, March 23 2015, available online at http://niskanencenter.org/wp-content/uploads/2015/03/The-Conservative-Case-for-a-Carbon-Tax1.pdf.

US Interagency Working Group on Social Cost of Carbon (IWG) (2013) "Technical Support Document: Technical update of the social cost of carbon for regulatory impact analysis Under Executive Order 12866." United States Government.

7 FIGURES

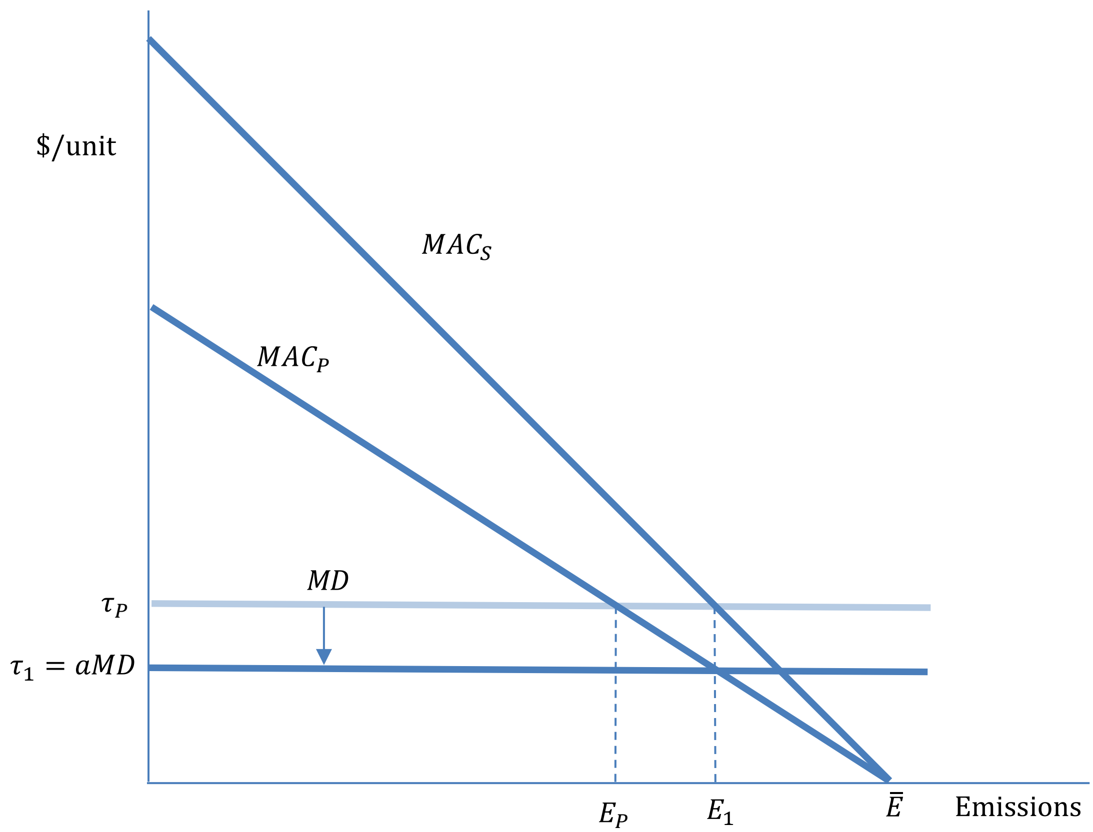

FIGURE 1: The classical Pigovian tax τ_P and the Sandmo result τ_1.

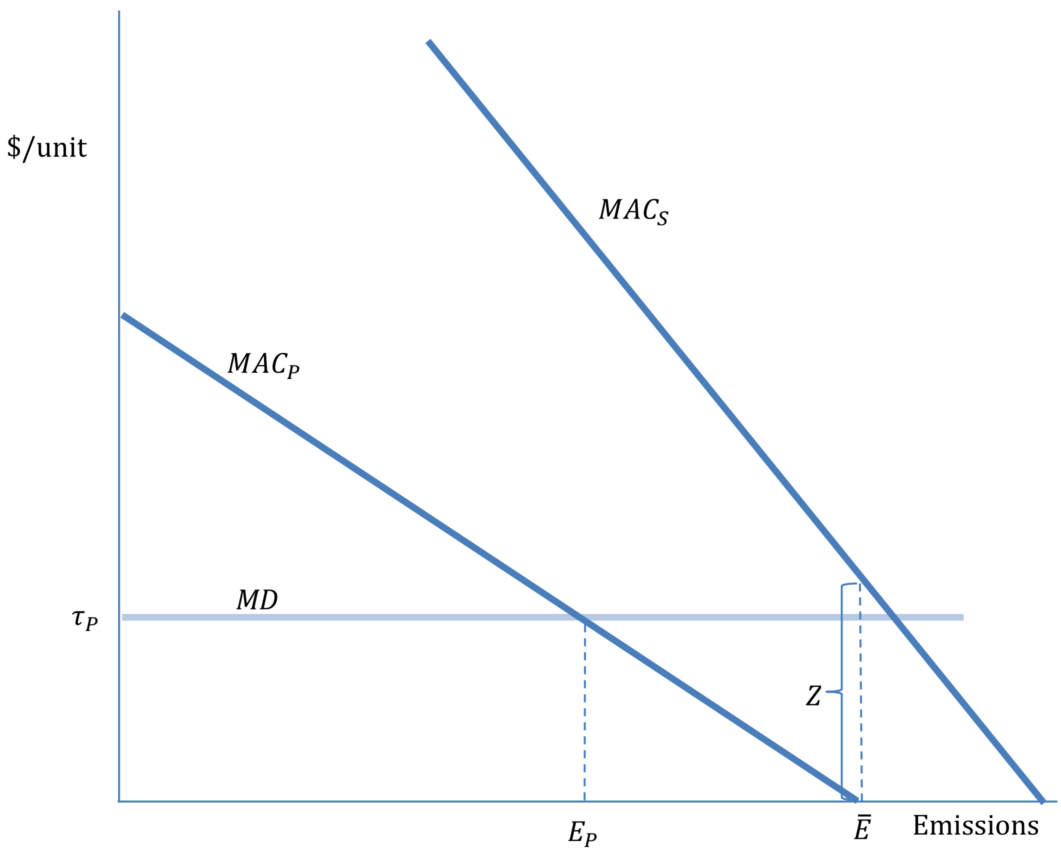

FIGURE 2: The classical Pigovian tax τ_P and the damage threshold Z.

8 APPENDIX

NBC+LME+EME+GME implies GBC.

Equations (2), (4), (7), (8), (9) and the definition of C imply

$$p_1X_1 + p_2X_2 + (p_E + \tau_C c)E_{hh} + w'H = (p_1F^1 - wL_1 - (p_E + \tau_C)E_1 + p_2F^2 - wL_2 - (p_E + \tau_C)E_2 + p_EE - wL_E + wL + wH) - \tau_y(\pi + wT).$$

Applying GME yields

$$\begin{aligned} p_1X_1 + p_2X_2 + p_EE_{hh} + \tau_C cE_{hh} + wH - \tau_Y wH \\ &= p_1X_1 - wL_1 - p_EE_1 - \tau_C cE_1 + p_2X_2 + p_2G - wL_2 - p_EE_2 - \tau_C cE_2 + p_EE - wL_E \\ &+ wL + wH - \tau_Y\pi - \tau_Y wL - \tau_Y wH. \end{aligned}$$

Canceling terms and collecting sums yields

$$p_E(E_{hh} + E_1 + E_2 - E) - w(L - L_1 - L_2 - L_E) = p_2G - \tau_Y(\pi + wL) - \tau_C cE.$$

LME plus EME the left hand side equals zero, yielding

$$p_2G = \tau_Y(\pi + wL) + \tau_C C. \blacksquare$$

Derivation of Equation (13**)**

Recall equation (14):

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - E_{hh}\frac{dp_E'}{d\tau_C} - H\frac{dw'}{d\tau_C} + \frac{dY'}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

Use $\frac{dY'}{d\tau_C} = (1 - \tau_Y)\frac{d\pi}{d\tau_C} - \pi\frac{d\tau_Y}{d\tau_C} + H\frac{dw'}{d\tau_C} + L\frac{dw'}{d\tau_C}$ and note that because w is the numeraire, $\frac{dw'}{d\tau_C} = -w\frac{d\tau_Y}{d\tau_C}$ to obtain

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - X_2\frac{dp_2}{d\tau_C} - cE_{hh} - H\frac{dw'}{d\tau_C} + H\frac{dw'}{d\tau_C} + L\frac{dw'}{d\tau_C} + (1 - \tau_Y)\frac{d\pi}{d\tau_C} - \pi\frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}$$

$$= -X_1 \frac{dp_1}{d\tau_C} - X_2 \frac{dp_2}{d\tau_C} - cE_{hh} - wL \frac{d\tau_Y}{d\tau_C} + (1-\tau_Y)\frac{d\pi}{d\tau_C} - \pi \frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dE}{d\tau_C}$$

$$= -X_1 \frac{dp_1}{d\tau_C} - X_2 \frac{dp_2}{d\tau_C} - cE_{hh} - \frac{d\tau_Y}{d\tau_C}(\pi + wL) + (1-\tau_Y)\frac{d\pi}{d\tau_C} - \frac{\delta N}{v_y}\frac{dE}{d\tau_E}.$$

When firms optimize inputs the envelope theorem implies $\frac{d\pi^i}{d\tau_C} = -cE_i$. Note also that $\pi + wL = B$, so $\frac{dB}{d\tau_C} = -c(E_1 + E_2) + w\frac{dL}{d\tau_C}$. Use these and equation (9)

$$\frac{d\tau_Y}{d\tau_C} = \frac{1}{B}\left(G\frac{dp_2}{d\tau_C} - \tau_Y \frac{dB}{d\tau_C} - \tau_C \frac{dC}{d\tau_C} - C\right)$$

to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1 \frac{dp_1}{d\tau_C} - X_2 \frac{dp_2}{d\tau_C} - cE_{hh} - B \times \frac{1}{B}\left(G\frac{dp_2}{d\tau_C} - \tau_Y\left(w\frac{dL}{d\tau_C} - c(E_1+E_2)\right) - \tau_C\frac{dC}{d\tau_C} - C\right) +$$

$$(1-\tau_Y)\frac{d\pi}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}$$

$$= -X_1 \frac{dp_1}{d\tau_C} - X_2 \frac{dp_2}{d\tau_C} - cE_{hh} - G\frac{dp_2}{d\tau_C} + \tau_Y\left(w\frac{dL}{d\tau_C} - c(E_1+E_2)\right) + \tau_C\frac{dC}{d\tau_C} + C + \frac{d\pi}{d\tau_C} - \tau_Y\frac{d\pi}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

$$= -X_1 \frac{dp_1}{d\tau_C} - (X_2 + G)\frac{dp_2}{d\tau_C} - cE_{hh} - c(E_1+E_2) + C + \tau_Y w\frac{dL}{d\tau_C} + \tau_C\frac{dC}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

Expand the derivative of labour as $\frac{dL}{d\tau_C} = \frac{\partial L}{\partial p_E'}c + \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_C} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_C} + \frac{\partial L}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}$. Use this, GME and the emissions function to reduce the above to

$$\frac{dW}{d\tau_C}\frac{1}{v_y} = -F^1\frac{dp_1}{d\tau_C} - F^2\frac{dp_2}{d\tau_C} + \tau_Y w\left(\frac{\partial L}{\partial p_E'}c + \frac{\partial L}{\partial p_1}\frac{\partial p_1}{\partial \tau_C} + \frac{\partial L}{\partial p_2}\frac{\partial p_2}{\partial \tau_C}\right) + \tau_Y w\left(\frac{\partial L}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}\right) + \tau_C\frac{dC}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C}.$$

The rest follows immediately. ■

Derivation of Equation (16)

Rearrange $\frac{dW}{d\tau_C}\frac{1}{v_y} = -F^1\frac{dp_1}{d\tau_C} - F^2\frac{dp_2}{d\tau_C} + \tau_Y w\frac{dL}{d\tau_C} + \tau_C\frac{dC}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC}{d\tau_C} = 0$ to get

$$\tau_C = \frac{\delta N}{v_Y} + \frac{1}{dC/d\tau_C}Q - \frac{\tau_Y R}{dC/d\tau_C}.$$

Using equation (8) we have $\tau_Y R = \left(\frac{p_2 G}{B} - \frac{\tau_C C}{B}\right)R$. Making the substitution yields

$$\tau_C = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC}{d\tau_C}}\,Q - \frac{Rp_2G}{B\frac{dC}{d\tau_C}} + \frac{RC}{B\frac{dC}{d\tau_C}}\tau_C$$

$$\tau_C\left(1 - \frac{RC}{B\frac{dC}{d\tau_C}}\right) = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC}{d\tau_C}}\,Q - \frac{Rp_2G}{B\frac{dC}{d\tau_C}}$$

$$\tau_C = \frac{\delta N}{v_Y}\left(\frac{\frac{dC}{d\tau_C}}{\frac{dC}{d\tau_C} - \frac{RC}{B}}\right)^{-1} + \left(\frac{Q}{\frac{dC}{d\tau_C}} - \frac{Rp_2G}{B\frac{dC}{d\tau_C}}\right)\left(\frac{\frac{dC}{d\tau_C}}{\frac{dC}{d\tau_C} - \frac{RC}{B}}\right)^{-1}.$$

The rest follows immediately.

Interpretation of Equation (17**)**

Multiply the top and bottom by τ_C then note that $dC/d\tau_C = cdE/d\tau_C$ and $\partial L/\partial\tau_C = -\partial H/\partial\tau_C$, which yield

$$a = \frac{c\tau_C\frac{dE}{d\tau_C}}{c\tau_C\frac{dE}{d\tau_C} + \tau_C\frac{C}{B}w\;\frac{\partial H}{\partial\tau_C}}.$$

Recall from equation (3) that $c\tau_C$ is the wedge between the supply price of energy and the marginal willingness to pay, hence the numerator is the welfare loss associated with a reduction in energy consumption due to an incremental increase in the emissions tax for the purpose of funding additional government spending. The same term appears in the denominator. The second term is the decline in leisure due to the emission tax weighted by $\tau_C Cw/B$. To understand this term, note that solving the GBC for τ_Y would break it down to two components: $p_2 G/B$ and $-\tau_C C/B$. The first is the portion required to cover government spending and the second is the offsetting reduction permitted by emission tax revenues. If government spending were zero but marginal damages necessitated $\tau_C > 0$ we could use the emission tax revenue to subsidize labour at the rate $-\tau_C C/B$. This represents the opportunity cost of needing to fund G. Hence the decline of leisure is weighted by the nominal wage rate times the portion of the income tax rate that represents the opportunity cost of needing to fund the government. Consequently, the denominator of a is the marginal (with respect to τ_C) opportunity cost of financing government spending through τ_C, and the inverse of a is this amount relative to the direct economic cost of the emission tax increase, giving a an interpretation similar to the inverse-MCPF weights found in previous models.

Derivation of Equation (19)

The derivative of W with respect to the emissions constraint yields

$$\frac{dW}{d\hat{C}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{C}} - X_2\frac{dp_2}{d\hat{C}} - H\frac{dw'}{d\hat{C}} + \frac{dY'}{d\hat{C}} - \frac{\delta N}{v_y}.$$

Since the policy does not raise revenue the GBC is $p_2 G = \tau_Y B$ which implies $-B\frac{d\tau_Y}{d\hat{C}} = (-G\frac{dp_2}{d\hat{C}} + \tau_Y\frac{d\pi}{d\hat{C}} + \tau_Y w\frac{dL}{d\hat{C}})$. Also note that $\frac{dY'}{d\hat{C}} = \frac{d\pi}{d\hat{C}}(1-\tau_Y) - \pi\frac{d\tau_Y}{dC} + T\frac{dw'}{d\hat{C}}$. Combining these yields:

$$\frac{dW}{d\hat{C}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{C}} - X_2\frac{dp_2}{d\hat{C}} - H\frac{dw'}{d\hat{C}} + H\frac{dw'}{d\hat{C}} + \frac{d\pi}{d\hat{C}}(1-\tau_Y) - \pi\frac{d\tau_Y}{d\hat{C}} + L\frac{dw'}{d\hat{C}} - \frac{\delta N}{v_y}$$

$$\Rightarrow \frac{dW}{d\hat{C}}\frac{1}{v_y} = -X_1\frac{dp_1}{d\hat{C}} - X_2\frac{dp_2}{d\hat{C}} - \tau_Y\frac{d\pi}{d\hat{C}} - \pi\frac{d\tau_Y}{d\hat{C}} - Lw\frac{d\tau_Y}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right)$$

$$= -X_1 \frac{dp_1}{d\hat{C}} - X_2 \frac{dp_2}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right) - B \frac{d\tau_Y}{d\hat{C}} + -\tau_Y \frac{d\pi}{d\hat{C}}$$

$$= -X_1 \frac{dp_1}{d\hat{C}} - X_2 \frac{dp_2}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right) - G \frac{dp_2}{d\hat{C}} + \tau_Y w \frac{dL}{d\hat{C}}$$

$$= -F^1 \frac{dp_1}{d\hat{C}} - F^2 \frac{dp_2}{d\hat{C}} + \tau_Y w \frac{dL}{d\hat{C}} + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right)$$

Expand the derivative of labour to obtain $\frac{dL}{d\hat{C}} = \frac{\partial L}{\partial p_E'} \frac{\partial p_E'}{\partial \hat{C}} + \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial \hat{C}} + \frac{\partial L}{\partial p_2} \frac{\partial p_2}{\partial \hat{C}} + \frac{\partial L}{\partial w'} \frac{\partial w'}{\partial \hat{C}}$. Then

$$\frac{dW}{d\hat{C}} \frac{1}{v_y} = -F^1 \frac{dp_1}{d\hat{C}} - F^2 \frac{dp_2}{d\hat{C}} + \tau_Y w \left(\frac{\partial L}{\partial p_E'} \frac{\partial p_E'}{\partial \hat{C}} + \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial \hat{C}} + \frac{\partial L}{\partial p_2} \frac{\partial p_2}{\partial \hat{C}} + \frac{\partial L}{\partial w'} \frac{\partial w'}{\partial \hat{C}}\right) + \left(\frac{d\pi}{d\hat{C}} - \frac{\delta N}{v_y}\right).$$

Set this equal to zero and use $\frac{d\pi}{d\hat{C}} = MAC_p$ and $\frac{\delta N}{v_y} \equiv MD$ to obtain

$$MAC_p = MD + F^1 \frac{dp_1}{d\hat{C}} + F^2 \frac{dp_2}{d\hat{C}} - \tau_Y w \left(\frac{\partial L}{\partial p_E'} \frac{\partial p_E'}{\partial \hat{C}} + \frac{\partial L}{\partial p_1} \frac{\partial p_1}{\partial \hat{C}} + \frac{\partial L}{\partial p_2} \frac{\partial p_2}{\partial \hat{C}}\right) - \tau_Y w \frac{\partial L}{\partial w'} \frac{\partial w'}{\partial \hat{C}}$$

which corresponds to Equation (19). ■

Derivation of Equation (23**)**

Variables modified to take account of the change in their initial values due to the partial regulation are denoted with $\sim$. The derivative of W with respect to the emissions constraint yields

$$\frac{dW}{d\tau_C} \frac{1}{v_y} = -X_1 \frac{dp_1}{d\tau_C} - cE_{hh} - H \frac{dw'}{d\tau_C} + \frac{d\tilde{Y}'}{d\tau_C} - \frac{\delta N}{v_y} \frac{dC_1}{d\tau_C}.$$

Substitute in $\frac{d\tilde{Y}'}{d\tau_C} = (1 - \tau_Y) \frac{d\tilde{\pi}}{d\tau_C} - \tilde{\pi} \frac{d\tau_Y}{d\tau_C} + H \frac{dw'}{d\tau_C} + \tilde{L} \frac{dw'}{d\tau_C}$ to obtain

$$\frac{dW}{d\tau_E} \frac{1}{v_y} = -X_1 \frac{dp_1}{d\tau_C} - cE_{hh} - H \frac{dw'}{d\tau_C} + H \frac{dw'}{d\tau_C} + \tilde{L} \frac{dw'}{d\tau_C} + (1 - \tau_Y) \frac{d\tilde{\pi}}{d\tau_C} - \tilde{\pi} \frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y} \frac{dC_1}{d\tau_C}$$

$$= -X_1 \frac{dp_1}{d\tau_C} - cE_{hh} - w\tilde{L}\frac{d\tau_Y}{d\tau_C} + (1-\tau_Y)\frac{d\tilde{\pi}}{d\tau_C} - \tilde{\pi}\frac{d\tau_Y}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1 \frac{dp_1}{d\tau_C} - cE_{hh} - \frac{d\tau_Y}{d\tau_C}(\tilde{\pi} + w\tilde{L}) + (1-\tau_Y)\frac{d\tilde{\pi}}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1 \frac{dp_1}{d\tau_C} - cE_{hh} - cE_1 - c\tilde{E}_2 - \frac{d\tau_Y}{d\tau_C}\tilde{B} - \tau_Y\frac{d\tilde{\pi}}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}.$$

Following similar steps as before while noting that $\tilde{p}_2$ does not change in response to the emission tax, we have $-\tilde{B}\frac{d\tau_Y}{d\tau_C} = \tau_Y\frac{d\tilde{B}}{d\tau_C} + \tau_C\frac{dC_1}{d\tau_C} + \tilde{C}$ and $\frac{d\tilde{B}}{d\tau_C} = \frac{d\tilde{\pi}}{d\tau_C} + w\frac{dL_1}{d\tau_C}$. Substitute these in to obtain

$$\frac{dW}{d\tau_E}\frac{1}{v_y} = -X_1\frac{dp_1}{d\tau_C} - \tilde{C} + \tilde{C} + \tau_Y\frac{d\tilde{B}}{d\tau_C} + \tau_C\frac{dC_1}{d\tau_C} - \tau_Y\frac{d\tilde{\pi}}{d\tau_C} - \frac{\delta N}{v_y}\frac{dC_1}{d\tau_C}$$

$$= -X_1\frac{dp_1}{d\tau_C} + \tau_Y\frac{d\tilde{\pi}}{d\tau_C} + \tau_Y w\frac{dL_1}{d\tau_C} - \tau_Y\frac{d\tilde{\pi}}{d\tau_C} + \frac{dC_1}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right)$$

$$= -X_1\frac{dp_1}{d\tau_C} + \tau_Y w\left(\frac{\partial L_1}{\partial p_E'}c + \frac{\partial L_1}{\partial p_1}\frac{\partial p_1}{\partial \tau_C}\right) + \tau_Y w\frac{\partial L_1}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C} + \frac{dC_1}{d\tau_C}\left(\tau_C - \frac{\delta N}{v_y}\right)$$

Set this to zero to obtain $\tilde{\tau}_C = \frac{\delta N}{v_y} + \frac{\tilde{Q}}{dC_1/d\tau_C} - \frac{\tau_Y\tilde{R}}{dC_1/d\tau_C}$ where $\tilde{Q} = F_1\frac{dp_1}{d\tau_C} - \tau_Y w\left(\frac{\partial L_1}{\partial p_E'}c + \frac{\partial L_1}{\partial p_1}\frac{\partial p_1}{\partial \tau_C}\right)$ and $\tilde{R} = \tau_Y w\frac{\partial L_1}{\partial w'}\frac{\partial w'}{\partial \tau_Y}\frac{\partial \tau_Y}{\partial \tau_C}$. The modified GBC yields $\tau_Y\tilde{R} = \left(\frac{p_2 G}{\tilde{B}} - \frac{\tau_C\tilde{C}}{\tilde{B}}\right)\tilde{R}$. Substituting as before yields

$$\tilde{\tau}_C = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC_1}{d\tau_C}}\tilde{Q} - \frac{\frac{p_2 G}{\tilde{B}}}{\frac{dC_1}{d\tau_C}}\tilde{R} + \tilde{\tau}_C\frac{\frac{\tilde{C}}{\tilde{B}}\tilde{R}}{\frac{dC_1}{d\tau_C}}$$

$$\tilde{\tau}_C\left(1 - \frac{\frac{\tilde{C}}{\tilde{B}}\tilde{R}}{\frac{dC_1}{d\tau_C}}\right) = \frac{\delta N}{v_Y} + \frac{1}{\frac{dC}{d\tau_C}}\tilde{Q} - \frac{\frac{p_2 G}{\tilde{B}}}{\frac{dC_1}{d\tau_C}}\tilde{R}$$

which solves to equation (23). ■